International Journal of Information Science & Technology – iJIST, ISSN : 2550-5114
Vol. 4 - No. 2 - July 2020

The non-cooperative game theory applied to telecommunication systems

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***Abstract*—In this paper, we outline various tools from the theory of non-cooperative games for studying competitive situations in telecommunication networks. We describe the mathematical tools while providing examples of various fields of telecommunication systems. In this paper, we study wireless systems in which mobile devices are autonomous in the choice of their communication configurations. This independence decision may involve, in particular the choice of the network access technology, the selection of the access point, the signal modulation, the frequency bands occupied, the power of the transmitted signal, etc. Typically, these configuration choices are made in order to maximize performance metrics specific to each terminal. Assuming that the terminals take their rational decisions to maximize their performance, game theory applies naturally to model the interactions between the decisions of different terminals. Specifically, the main objective of this paper is to study emission power control equilibrium strategies to satisfy energy efficiency considerations.**

***Index Terms*— Game theory, telecommunications system, Nash equilibrium, utility, non-cooperative games.**

1 .INTRODUCTION

Game theory is a central tool in many disciplines: in economics, biology, business and finance, road transport, marketing, political and social sciences, ecology and the environment, operational research, and many others. It also plays a role in military research in control theory, in telecommunication networks and computer networks **[1]**. But game theory research is not limited to the applications mentioned above. Indeed, the very foundations of game theory is a mathematical field still very active, which uses a lot of mathematical tools that often seem more sophisticated than the problem at hand. This is the case of algebraic logic, algebraic geometry, and the approach of viscosity solutions for differential equations appearing in dynamic games in continuous time. Game theory is probably the best known in the context of economics, especially since 1994, when the Nobel Prize in economics was attributed to researchers for their contribution to the analysis of equilibrium and non-cooperative game theory. Recall that before this, K. Arrow and J. HICS (1972) and Debreu (1983) received the Nobel Prize on their contribution to the theory of equilibrium, related to game theory **[2]**. In this paper, we present some facets of non-cooperative game theory that provide a framework for modeling and analysis for competitive situations in telecommunication networks.

The framework of stochastic games is particularly suited to this problem and allows us in particular to characterize the region of performance achievable for all power control strategies that lead to a state of equilibrium. When the number of game terminals is large, we use the theory of mean field games to simplify the study of the system. This theory allows us to study not the individual interactions between the terminals, but the interaction of each terminal with an average field representing the overall state of the other terminals. For a power control game, the convergence of dynamics of better responses to equilibrium points has been studied.

A few words on the structure of the paper: in section 2, we make a reminder about basic concepts in game theory. We highlight hierarchical aspects of decision making, as well as multi-criteria aspects. In Section 3, we introduce mathematical models to treat these games. Then, we present in section 4 basic tools that serve to answer the questions of the existence and the uniqueness of equilibrium. In Section 5, we discuss coordination problems in games by introducing the concept of correlated equilibrium. An evolutionary game, an approach to game theory that comes from mathematical biology, is described in Section 6, as well as its potential contribution to networks. Section 7 is devoted to conclusions.

II. BASICS OF GAME THEORY

Game theory focuses on situations where "players" or "agents" make decisions, each being aware that his or her earnings depend not only on his or her own decision, but also on decisions made by other players **[3]**. A player can make several decisions and he chooses one that will be the best for him. In mathematical terms, we translate the sentence "best for him" by introducing a function for each player that reflects his preferences, called "utility". The utility of a player may depend not only on its decisions but also those of all the other players. The utility is an increasing function with respect to preference: the value of a player is higher for a choice of decisions in relation to another if he prefers the first choice to another. The solution concept of a non-cooperative game is often the Nash equilibrium: it is a decision-making choice of all players such that none can take advantage (can not increase its utility function) by changing only (unilaterally) his decision **[4]**.

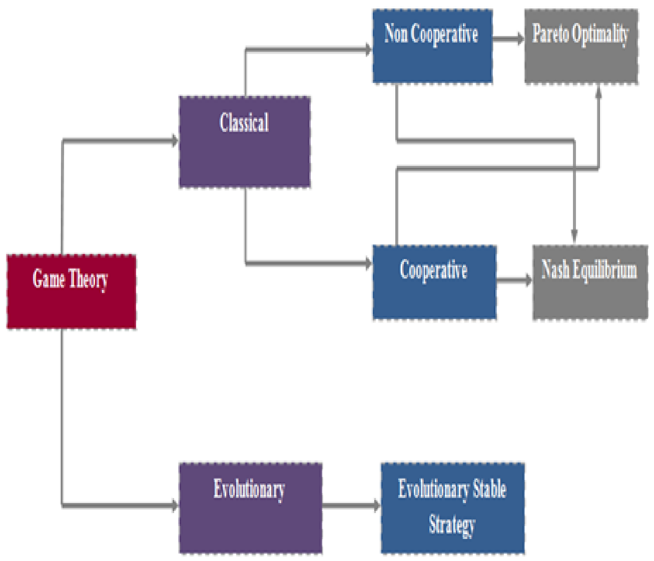

Figure 1: The perspectives of game theory

2.1 Hierarchical games:

Competitive situations can occur at several levels. For example, we could imagine that a service provider has several classes of service that are distinguished by the quality of each service (offered rate, time etc.) but also by the cost of the service. We can then identify a situation of non-cooperative games between subscribers. Indeed, the quality of service perceived by a subscriber may depend on the choices of each other subscriber **[5]**.

The balance that describes the decisions made by the subscribers will determine the gains of the service provider. The latter therefore has an interest in choosing the qualities of service it offers as well as their costs in a way to maximize its profits, and this taking into account the balance between subscribers that will be generated by its decisions. Quality of service choices impose well on network architecture choices as well as network management policies, which makes the problem relevant to network engineering.

The result of this situation of choosing the best decision at the supplier level that takes into account the reaction of subscribers is described by a balance called Stackelberg that we describe later. This is called bi-level optimization , we could imagine even more complex competition situations, where the demand, and therefore the earnings of a service provider, depends not only on the reaction of subscribers to the decisions of their provider, but also choices made by other competing service providers.

Let's call "agent" someone who makes decisions **[6]**. An agent in our example can be a subscriber or a service provider. However, we can imagine other levels that involve other agents. For example, a service provider is not necessarily the one that has the network belongs. In this case, we could have a representative agent of the owner network that can be confronted with pricing choice offered to service providers for the network resource allocation.

2.2 Multi-criteria aspect:

The approach of considering only one criterion that an agent wants to maximize is often not sufficient to describe the needs and behavior of agents. For example, for interactive voice over internet services, the perceived quality depends on the code used, the bit rate, the packet loss rate, the delay, and its variability **[7]**. A simplistic approach often used to deal with this multi-criteria aspect is to define a single criterion that takes into account several qualities of service. Other approaches that are more sensitive to each criterion consist in separating the criteria and defining equilibrium concepts that are sensitive to each of them: the Nash-Pareto equilibrium and the Nash equilibrium under constraints. We describe these concepts in the following section.

3. MATHEMATICAL MODELING AND EQUILIBRIUM

3.1 Nash equilibrium:

In the case where each user has only one evaluation criterion, the objective is to determine decisions for each of them, optimal in the sense of the Nash equilibrium concept **[8]**:

Suppose there are N subscribers for network access, each seeking to maximize a (unique) utility function. Let u_n denote the decision of subscriber n, and $J^n(u,x)$, its utility function. This function depends on the action u_n of the subscriber n, but also on the actions of all the other users; the variable $\mathrm{u}=(\mathrm{u}_1,\mathrm{u}_2,....,\mathrm{u}_\mathrm{N})$ is therefore the N-tuple of decisions taken by the N users, x is a parameter representing the architecture and the management policy of the network **[9]**. For architecture and a network policy, x, fixed, a N-tuple of decisions $\mathrm{u}^*(x)=(\mathrm{u}^*_1,\mathrm{u}^*_2,....,\mathrm{u}^*_\mathrm{N})$ is called Nash equilibrium if none of the N subscribers can improve its utility function by modifying only its decision. More precisely, for all $n\in\{1,2,....,N\}$, we have:

$$J^n(\mathrm{u}^*(x),x)=\max_{u_n} J^n(\mathrm{u}^*_1,....,\mathrm{u}^*_{\mathrm{n}-1},\mathrm{u}^*_\mathrm{n},\mathrm{u}^*_{\mathrm{n}+1},....,\mathrm{u}^*_\mathrm{N},x) \quad \textbf{(1)}$$

(Sometimes it's more natural to talk about minimizing a cost rather than maximizing a gain **[10]**. In such cases we will replace J^n with the symbol C^n.)

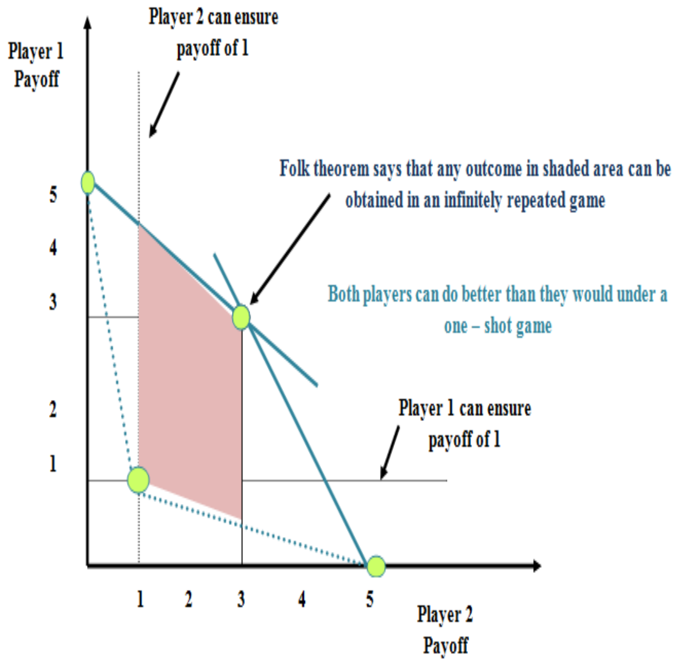

Figure 2: In infinity repeated game it is possible to obtain the cooperative result, unless interest rates are too high or discount rates too low.

As noted earlier, each user can seek to make decisions to maximize multiple criteria. We will consider two extensions to the Nash equilibrium concept in order to study the multicriteria case **[11]**. In this context, the utility function J^n of a user n is a vector, $J^n = (J^n{}_1, ..., J^n{}_k)$. We consider in this case two types of equilibrium

3.2 Multi-criterion equilibrium, or Pareto-Nash equilibrium:

The maximum in equation (1) is in the sense of Pareto.More precisely, we say that a vector x of dimension k dominates a vector y of the same dimension if for all $i = 1, ..., k$ we have $x_i \geq y_i$ with a strict inequality for at least one of i. In this case, we note x dom y. u^* is said multicriteria equilibrium if no subscriber can benefit (in the sense of the dom relationship) by unilaterally changing its decision-making: for each n, there is no political u_n which gives better performance the player n, that is to say, such as:

$$J^n(u^*{}_1, ..., u^*{}_{n-1}, u_n, u^*{}_{n+1}, ..., u^*{}_N, x) \,\mathrm{dom}\, J^n(u^*, x) \quad \textbf{(2)}$$

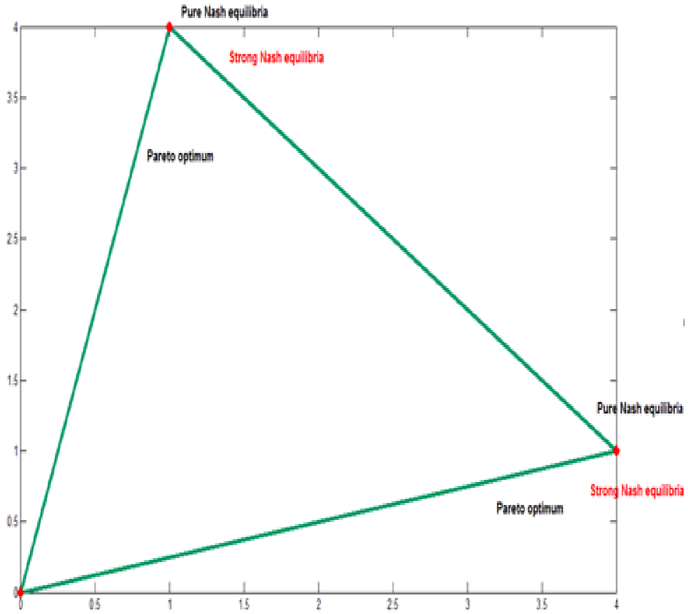

Figure 3: The game space (pure Nash equilibrium)

3.3 Nash equilibrium with constraints:

In this case, it is assumed that a player seeks to maximize the $J_1{}^n$ criterion while maintaining his other criteria $J_2{}^n ....J_k{}^n$ within certain limits. It is therefore a question of looking for a Nash equilibrium under constraints **[12]**. The concept of constrained optimization in the multi-criteria case is natural in a A.T.M architecture (Asynchronous Transfer Mode) where subscribers express their QoS demands by constraints on time, the loss rate, etc ... For example, an interactive audio application is insensitive to a delay as it remains lower about 100 msec. An audio application could therefore seek to minimize the loss rate while trying to enforce a constraint on time.

Let $\prod_n(x) = (u : J_i{}^n(u, x) \leq \mathrm{V}_i^n, i = 2, ..., \mathrm{k}_n)$ denote the set of N-tuples of actions of N subscribers respecting the $k_n - 1$ constraints of subscriber n, where V_i^n is the bounds defining these constraints. u^* is then a Nash equilibrium under constraints if for all n, $u^* \in \prod_n(x)$, and if more **[13]**:

$$J^n(u^*, x) = \max_{u_n} J^n(u^*{}_1, ..., u^*{}_{n-1}, u_n, u^*{}_{n+1}, ..., u^*{}_N, x) \quad \textbf{(3)}$$

Where it is restricted in maximizing to u_n such that:

$$(u^*{}_1, ..., u^*{}_{n-1}, u_n, u^*{}_{n+1}, ..., u^*{}_N) \in \prod_n(x) \quad \textbf{(4)}$$

3.4 Multicriteria hierarchical optimization:

We consider that the operator (administrator or network designer, service provider) also seeks to maximize a certain number of criteria. These criteria may include, among other things, subscriber criteria (the operator probably having an interest in satisfied subscribers), but also criteria for the efficient use of resources as well as more purely economic criteria [14]. We denote $R(u,x)$ the utility function of the operator; this depends, on the one hand, on the architecture and management policy of the network (through x), and on the other hand, the u behavior of the subscribers.

In the case where the equilibrium $u^*(x)$ defined in sub-section 3 exists and is unique, the objective of the network operator is to determine x which maximizes its evaluation functions (utility), assuming that subscribers choose equilibrium u^* shares. In other words, the objective of the network operator is to find x^* which verifies [15]:

$$R(u^*(x^*), x^*) = \max_x R(u^*(x), x) \quad \textbf{(5)}$$

When the subscribers and the operator of the network each have a scalar J^n and R evaluation function (a single criterion taken into account for each), the set $(u^*(x^*), x^*)$ is a Stackelberg Equilibrium. We can consider the more general case where J^n and R are vectors [16].

In this case, equation (4) means that there is no x as $R(u^*(x), x) \operatorname{dom} R(u^*(x^*), x^*)$. In the case where there exists a set $U^*(x)$ which contains several balances for the subscribers (which is often the case when it is a Pareto-Nash equilibrium for the subscribers), the network aims to ensure the best return for any possible equilibrium, that is to say, seek x^* that verifies [17]:

$$R(u^*(x^*), x^*) = \max_x \min_{u^* \in U^*} R(u^*(x), x) \quad \textbf{(6)}$$

We also consider the case where several operators are competing on the network. If the behavior of the subscribers was fixed, one would return to the framework of the games defined in the preceding subsections, the players being this time the operators [18]. Again, the concept of multi-objective optimization solution would be one of the extensions in section 3.2 or 3.3.

In case the subscribers can also choose their behavior in the network, it comes back to the complex situation of a two-level game, or hierarchical play. Taking into account the reactions, $u^*(\mathrm{x})$, of the N subscribers to the decisions, $\mathrm{x} = (x_1, x_2, \ldots, x_M)$ of the M operators, the concept of solution would be an extension of the equation (4) in the form

$$\mathrm{R}^{\mathrm{i}}(u^*(\mathrm{x}^*), \mathrm{x}^*) = \max_{x^i} R_i(u^*(\mathrm{x}^*_{-i}, x_i), \mathrm{x}^*_{-i}; x_i) \quad \textbf{(7)}$$

Where $\mathrm{x}^*_{-i} = (x^*_1, \ldots, x^*_{i-1}, x_i, x_{i+1}, \ldots, x_M)$, R^{i} represents the utility (scalar in the monocriterion or vector case in the multicriterion case) of the operator i, and x_i its decisions [19].

4. EXISTENCE AND UNIQUENESS OF EQUILIBRIUM

4.1 Concave games:

Consider games where the set of strategies is convex in R^m for $m \geq n$. In general, the set of $U_i(u^{-i})$ strategies available for any i player depends on the actions of other players. We have seen such dependence in the formulation (3). To illustrate the need for such a definition, consider the following example. Consider the uplink in a cellular network with a base station and mobile N. Let $\mathrm{u_i}$ be the received power at the base station from the mobile i. The signal / noise ratio at the base station corresponding to the mobile i is [20]:

$$SIR_i = \frac{\mathrm{u_i}}{N_0 + \sum_{j \neq i} \mathrm{u}_j} \quad \textbf{(8)}$$

Where N_0 is the thermal noise power at the base station. We assume that the mobile i needs to obtain a signal-to-noise ratio SIR_i greater than or equal to a threshold γ_i (to guarantee a sufficiently low loss rate and a sufficient transmission rate). Imagine that the mobile i aims to minimize its power $\mathrm{u_i}$. Given the transmission powers u_j of other players, the available strategies of the mobile i are given by

$$U_i(u^{-i}) = \left\{ \mathrm{u_i} : \mathrm{u_i} \geq \gamma_i (N_0 + \sum_{j \neq i} \mathrm{u}_j) \right\} \quad \textbf{(9)}$$

Two particular cases of dependence between a player's strategies and the strategies of other players are studied in [21]:

- Common Constraints: There is a convex set of U policies. It is said that a policy $u = (u_1, \ldots, u_n)$ satisfies the constraints if it belongs to U. In other words, for every i, $U_i(u^{-i}) = ((u_i, u^{-i}) \in U)$.

- Orthogonal constraints: this is the case where $U_i(u^{-i})$ does not depend on player i

In both cases, we have the following result, has the following result under the assumption of convexity on the sets of policies.

Theorem: Suppose J^n is concave relative to $u^i \in U_i$ and it is continuous with respect to u^j, for all $j \neq i$. So the game has equilibrium **[22]**.

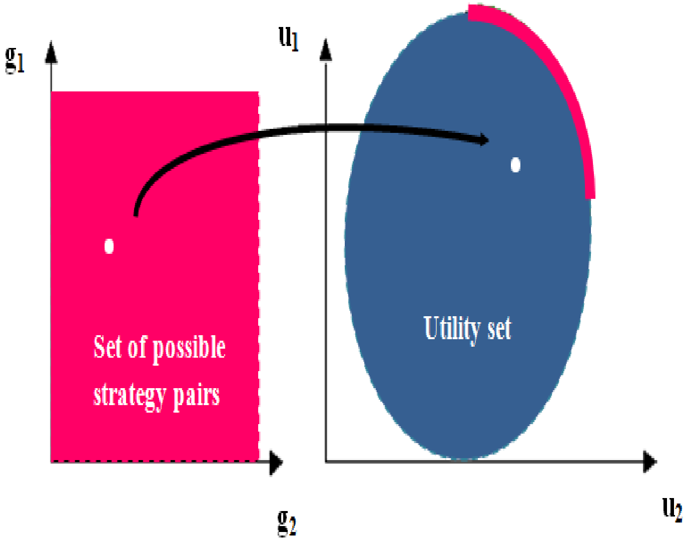

Figure 4: Bargaining Problem overall utility

4.2 Games with a finite number of actions:

Consider the case of N non-cooperative players each having a single criterion to maximize. When each agent has a finite number of actions, there may be no equilibrium at all. We often widen the action class players: instead of choosing one of the stocks, an agent chooses a probability on the stock that is available to him. Then, instead of focusing on maximizing income, the agent maximizes the expected value of his income. In equilibrium there may be several players who make random choices. These choices are made independently of each other **[23]**.

When the game has a finite number of actions, it can be represented as a matrix game of dimension N. Each multi-strategy $u_1,....,u_N$ associates an element of the matrix.

This element contains the N values corresponding to the utilities $J^i(u_1,....,u_N)$ where $\mathrm{i}=1,....,\mathrm{N}$. Note that in matrix games, the utility of the strategy is concave (and even linear) with respect to the policy used (because the mathematical expectation is a linear operator). In addition, being linear, the utility of a player is continuous with other players' policies.

5. COORDINATION IN NON-COOPERATIVE GAMES

As we have already seen, there are often Nash equilibrium that use mixed policies, that is, policies that use a random choice between several strategies. In the Nash equilibrium, when several players make random choices, these choices are made independently of each other. However, this independence can lead to balances that give weak utilities **[24]**.

5.1 The identity of coordination:

To date, game theory does not provide a precise framework for linking the two concepts of coalitions and networks. Generally, a coalition is defined as a set of players. This is to be contrasted with the other players, who are not in the coalition. Nothing is specified as to the nature of the relationship between the coalition members, or that they have with the players out of the coalition. In most cooperative games, a coalition (or players belonging to this coalition) does not have interaction with players outside the coalition. However, in other situations, such as cartels industrial economy, there may be interactions between a coalition and other players **[25]**.

A network, instead of a coalition, specifies relationships that each player has with other players. This would imply that in a coalition, each player is bound to the other members of his coalition in the same way that the other members are related to each other. So a coalition would not only be a grouping of agents, for example in collegiate form, but would also be equivalent to a complete network, that is to say a network such that each player in a coalition is linked to all other agents of his coalition. Such a result would it be valid when considering such a non-cooperative game of network formation. More generally, the question is whether the coalitions and networks can be treated in the same frame, or if we can provide a unified approach to the problems of formalization of coalitions and networks. In network training games, the approach traditionally used is that proposed by R. Myerson [24 and R. R. Aumann and Myerson . If by cons we want to study the coalitions in cooperative games, the "traditional" value is the Shapley value. [26] .

In this case, we easily obtain the result (3) that a coalition is nothing more than a complete network such that each agent in the coalition is connected to all the other agents in the network, is not attached to any non-member of the coalition agent. Thus, one could be tempted to say that a coalition is only a particular case of a network, and thus make a link between coalitions on the one hand and networks on the other hand. It would be imprudent to stay with this assertion. Indeed, the identity between full network and coalition can be easily demonstrated as "wobbly".

Consider a cooperative game (N,v), where N is a set of players, and v is the characteristic function. The latter assigns a value to each coalition, that is, for each subset of N

(including the empty set).The value of a coalition can be interpreted as a monetary sum available to the coalition, which the members of this coalition have to share among themselves (4).The solution to a cooperative game then consists of a value that determines for each player its share.

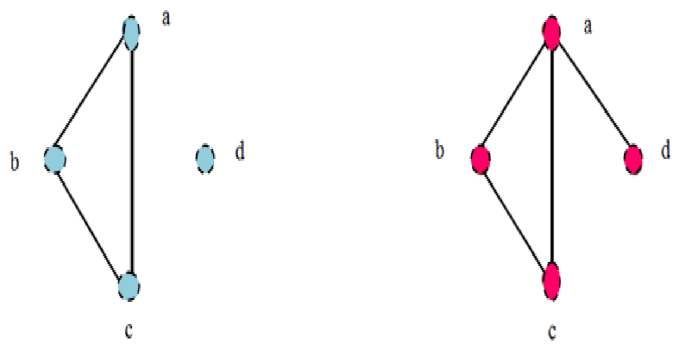

Figure 5: The g1 and g2 networks

As we have said, when we analyze a cooperative game where agents can be grouped into a network, the value usually used is Myerson's value. Here, a network is to be interpreted as a graph, whose vertices represent the players and stop their links with each other. Let $\mu(N,\nu,g)$ be the value of Myerson when we have a network g (and ν any characteristic function). If on the contrary we analyze the coalitions, the value usually used is the value of Shapley. Let $\varphi(N,\nu)$ be this value (5). A network is said to be complete if all the players are directly connected to each other. R. Myerson showed that if g_N is a complete network, then we have:

$$\mu(N,\nu,g_N)=\varphi(N,\nu)$$

In other words, when the network is complete the Shapley value and the value of Myerson coincide. At first glance; we can conclude that networks and coalitions can be treated in the same framework, in which a coalition is nothing more than a complete network. While remaining within this framework (Myerson's value and Shapley's value), we will now show that this conclusion is erroneous.

Let the network g_1 where the set of players is $N=\{\mathrm{a},b,c,d\}$, such that a, b and c are directly connected to each other, and the player d is isolated. This network is shown in Figure 4. In this case, it is easily shown that for $N'=\{\mathrm{a},b,c\}$,

$$\mathrm{i}=\mathrm{a,b,c}\ \mu_{\mathrm{i}}(\mathrm{N},\nu,\mathrm{g}_1)=\mu_{\mathrm{i}}(\mathrm{N'},\nu,\mathrm{g}_1)=\varphi_i(N',\nu) \quad \textbf{(10)}$$

where μ_{i} and φ_i are respectively the value of Myerson and the value of Shapley of the player i. A priori, the results of equations (1) and (2) confirm the idea that a coalition is a complete network. Consider now the network g_2 (in figure 4), which is identical to the network g_1 except that there is now a link between the player a and the player d. It is then very easy to find a characteristic function ν such that equation (2) is no longer satisfied, even if the players a, b and c are symmetrical **[27]**.

We could object (with reason) that in the network g_2 the players a, b and c no longer have the same position relative to the player d. But this does not detract from the fact that $\{\mathrm{a},b,c\}$ can no longer be considered as a coalition.

Moreover, we can take solutions other than the value of Myerson and the value of Shapley (6), such that, even with a complete network regrouping all the players, an equivalent of the equation (1) cannot be held true.

Certainly, the discussion we have just conducted may seem obvious to the specialists of cooperative games. However, it is instructive for the following reason. The theory of cooperative games is certainly the area of analysis where networks and coalitions were most studied. Despite its simplicity, the cooperative game theory does not allow us to develop a clear relationship between a coalition and a network. Indeed, we can not say if the network g_2 agents a, b and c form a coalition **[28]**.

In addition, we have not dealt with more complex cases. For example, we can imagine that there is a hierarchy within a coalition, that is, the coalition is described as a network. Another situation that we have not studied here is that the networks where relationships between players are specific players. In such a situation, the relationship that could connect two players, for example i and j, is not the same as the relationship between two other players h and k. In the networks g_1 and g_2 of figure (4), the relations between the players are all of the same nature **[23]**.

In fact, the literature has taken two different directions to build a link between networks and coalitions. There are two possible ways to model the networks. The first is to start from a cooperative game. The value of a network is derived from the values of the coalitions. The second approach consists in starting from a value on the networks, thanks to the non-anonymity principle. However, these two approaches do not make it easy to move from coalitions to networks and vice versa.

5.2 Cooperation and coordination:

In the previous section, we saw that game theory offers few tools to jointly study the networks and coalitions. This gap is mainly due to a lack of consensus and conciseness in defining what a coalition or network is. Suppose now that a coalition is defined (loosely) as a set of players such as **[22]**:

- All players in the coalition have the same importance.
- No player in a coalition has any relationship with non-members of his coalition of the same nature as he has with other members of the coalition.

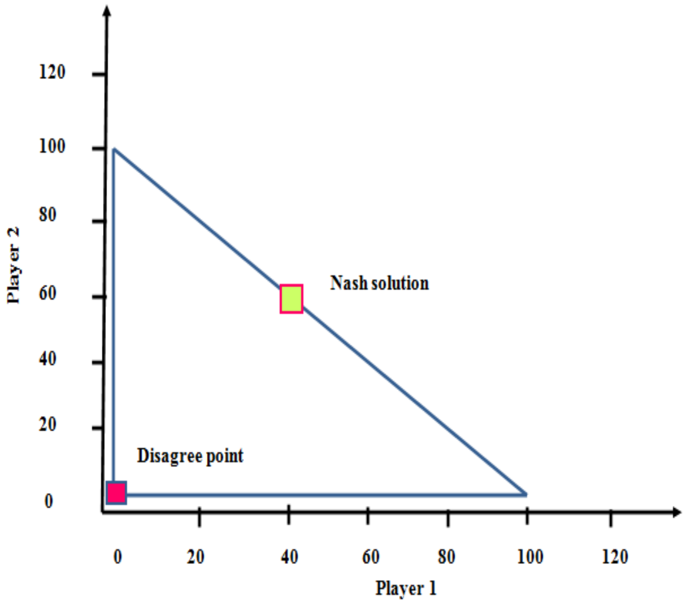

Figure 6: Nash Negotiation

We can observe that these two points mean that when we consider a set of players forming coalitions, they form a partition of all the players. This is also the approach generally adopted by most authors working on coalition formation problems. For this, consider a coalition problem in a non-cooperative game in strategic form. For simplicity, assume that coalitions are already formed. In non-cooperative games, the problem is to determine what the strategies are played by the players **[18]**. The existence of coalitions means that strategic decisions are made at the coalition. According to the point above, a decision criterion for coalitions can be that of Pareto dominance. The coalition containing only two members, i and j. Suppose that the strategy of the other player-coalitions is fixed and that i and j can only choose between the strategies a, b, c and d, such that the payments of i and j are:

$$a \to (2,2) \qquad b \to (3,2)$$

$$c \to (1,4) \qquad d \to (1,1)$$

Where the first digit is the gain of i.In this case, the "best answer" of coalition $\{i, j\}$ is multiple: either strategy b or strategy c. The a and d strategies are dominated Pareto.

To summarize, we have a non-cooperative game with n players, where some players are grouped into coalitions and strategic choices of the players in each coalition are on Pareto dominance criterion. If a player is alone, we can consider that he is in a coalition of which he is the only member and therefore his criterion of choice will be to maximize his gain. So we have a game where coalitions play together. The way to see such a game is to consider each coalition as a "meta-player".

The meta-game will be the game where the meta-players play with each other. As some may represent coalitions with two or more players, the gains of these meta-players are multi-dimensional vectors, and not a real. At first glance, the game we just built does not offer any methodological difficulty and seems to be clearly and completely defined **[26]**.

We can see that the definition of our game is incomplete when we seek to determine the existence of equilibrium. As the game we are considering is any, we must take into account the equilibrium in mixed strategies. The methodological mistake we make here: we need to clarify the nature of mixed strategies. The existence of equilibrium in a non-cooperative game where some players can be grouped within a coalition. According to the definition we take for the latter, their result may be correct or erroneous. Thus, the exact definition of strategies is crucial for the existence of equilibrium in a game in general.

Let $(N, \sum, u)$ be the non-cooperative game, where N is the set of players, $\sum = \times i \in N \sum_i$ the set of player strategies where $\sum_i$ is the set of strategies of the player i and $u = (u_i) i \in N$ the player's payoff functions. The set of mixed strategies of player i (isolated) is defined in the usual manner, that is to say it is all probability measures on $\sum_i$, which we denote $\Delta(\sum_i)$.

A first way is to say that the set of mixed strategies of the coalition S, which we denote $\Delta^1(\sum_S)$ is the Cartesian product of the sets of mixed strategies:

$$\Delta^1(\textstyle\sum_S) = \times_{i \in S} \Delta(\textstyle\sum_i) \qquad \textbf{(11)}$$

A second way is to say that the set of mixed strategies of the coalition S, which we will note $\Delta^2(\sum_S)$, is the set of probability measures on the set of pure strategies of the coalition S, which is the Cartesian product of pure strategies of coalition members **[21]**:

$$\Delta^2(\sum_S) = \Delta(\times_{i\in S}\sum_i) \qquad \textbf{(12)}$$

6. Evolutionary games:

Among the applications of game theory, we find the biology we need the concept of "evolutionary games". Assume large animal populations, or populations characterized by its behavior patterns (including among the population of a given animal). However, the interaction between different populations or between different behaviors is done through many local interactions between a small numbers of individuals **[19]**.

As we have already seen in other types of games, the behaviors adopted by the individuals during the interaction determine the utility (or the evaluation function) for each player. In the biological context, over the evaluation function is, the greater the chances of access to food are high, which can increase the reproduction rate of the individual. The novelties of these evolutionary games compared to non-cooperative games are summarized as follows:

- The concept of equilibrium solution called evolutionarily stable strategy (Evolutionary Stable Strategy - ESS), is different.
- These games often model a dynamic evolution of each population or each behavior according to the strategies used and obtained utilities.

6.1 Strategies:

Consider an evolutionary game where there are two strategies (or actions): action 1 and action 2 for each player. We allow the use of mixed strategies. We say that an entire population uses a mixed strategy $q \in [0,1]$ if the proportion of individuals in the population who use Action 1 is q, and the part of those who use Action 2 is $1-q$.

Note: the notion of strategy is also applicable to individuals **[20]**. We then consider the case where each individual is frequently in a situation of play (interaction) with other individuals, and we say that he follows a mixed strategy p if the fraction of times he has played 1 is p, and the fraction of times he used 2 is $1-p$. Assume that the stock picks of this individual are independently with probability p.

6.2 Utilities:

We define $J(p,q)$ the average utility of an individual who uses the p strategy while the other individuals he meets use the q strategy.

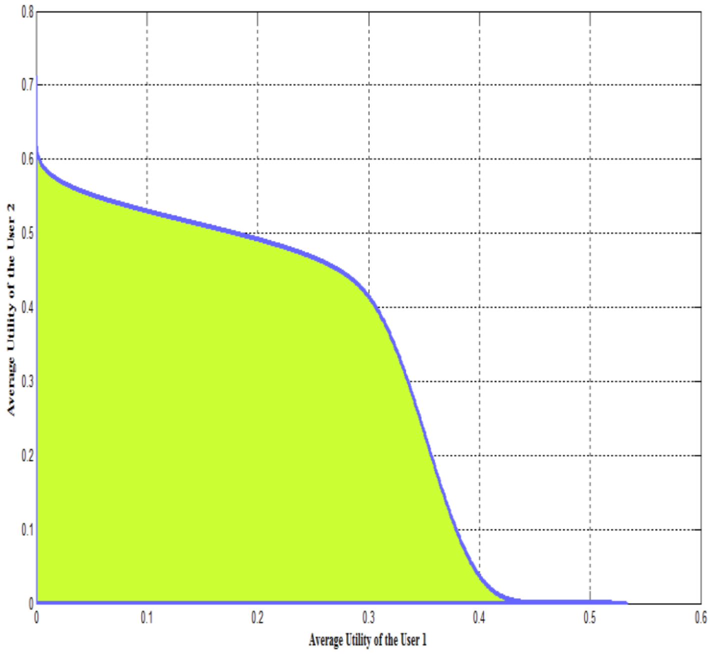

Figure 7: Achievable region and average utilities for 2 users

Suppose the population uses a mixed strategy q^* and a small fraction (called "mutations") adopts a different strategy (pure or mixed) p. If for any $p \neq q^*$ was

$$J(q^*,q^*) \succ J(p,q^*) \qquad \textbf{(13)}$$

Then the relative fraction of mutations in the population decreases (because their usefulness, which represents the rate of growth, is lower than that of the rest of the population) **[17]**. It is said that q^* is immune to mutations.

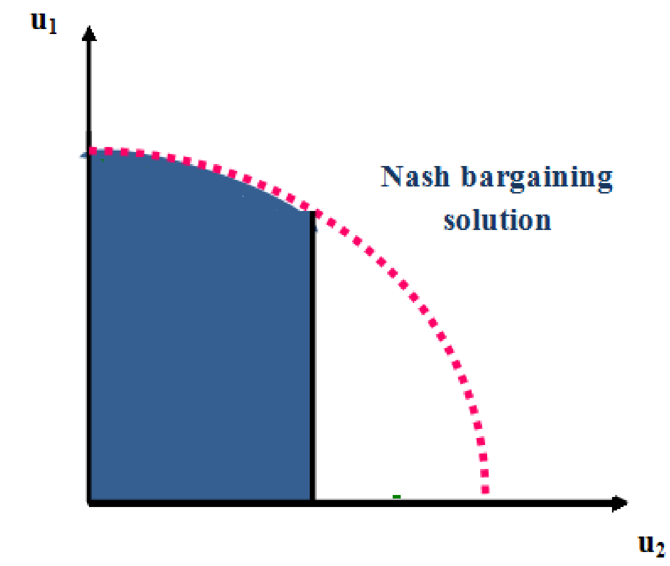

Figure 8: Nash solution to the cooperative game

6.3 New concept of equilibrium : ESS

If there are n pure strategies ($n = 2$ in our case) called $s_1,....,s_n$, then a sufficient condition for (7) is that

$$J(q^*,q^*) \succ J(s_i,q^*),\ s = 1,....,n \qquad \textbf{(14)}$$

In the particular case where we have

$$J(q^*,q^*) = J(p,q^*)$$
$$J(q^*,p) \succ J(p,p) \qquad \forall p \neq q^* \qquad \textbf{(15)}$$

We could say that the population that uses q^* is weakly immunized against the p behavior of mutations because, if the proportion of mutations increases, then we will often have individuals using the q^* strategy that will interact with the mutations. In this case, condition $J(q^*,p) \succ J(p,p)$ ensures that the growth rate of the original population dominates the mutants. q^* that satisfies (7) or (9) is called an evolutionarily stable strategy (ESS).Although the ESS has already been defined in the context of biological systems, it is well suited in the context of automation in general and in the control of networks in particular **[25]**.

6.4 Energy Administration:

We can adopt notions of biology, not only through the concept of evolutionary games, but also through applications related to energy management. In the context of biology, the survival, the life span and, consequently, the reproduction rate are related to the quantity of energy of an animal, and therefore to the behavior of the animal during a competition with other animals on resources. By analogy, we can expect that sensor networks that are designed with energy efficient strategies have a longer lifespan **[29]**.

6.5 Evolutionary games and networks:

At present, few works in telecommunication networks use evolutionary games, in the context of road traffic, through models that can also be used for telecom networks. Others also use the ESS in the context of power and rate control in wireless networks. The first advantage of evolutionary games and the notion of ESS equilibrium with respect to the Nash equilibrium is the robustness of the ESS. For a multi strategy u^* to be Nash equilibrium, only one player (any one) can benefit by changing his strategy. But with the Nash equilibrium, if several players change their decision, they may make a profit. The notion of ESS is more robust by allowing a whole fraction of a population (the mutants) to change their decision. So even when several players change their decision, they cannot take advantage of this change **[30]**. The second advantage of the evolutionary game framework is to propose dynamics that lead to the ESS, and that provide a justification for the use of ESS.

7. SIMULATIONS AND ANALYSIS OF THE RESULTS

In our simulations, we evaluate the average total throughput of the proposed schemes as a function of SNR in dB. The signal to noise ratio is an indicator of quality of the transmission of information that is generally expressed in decibels (dB). This is the power ratio between the maximum amplitude signal, determined by the maximum permissible value for the effects remain an allowable value. The background noise, non-significant information generally corresponding to this signal the output of the device in the absence of information to the input.

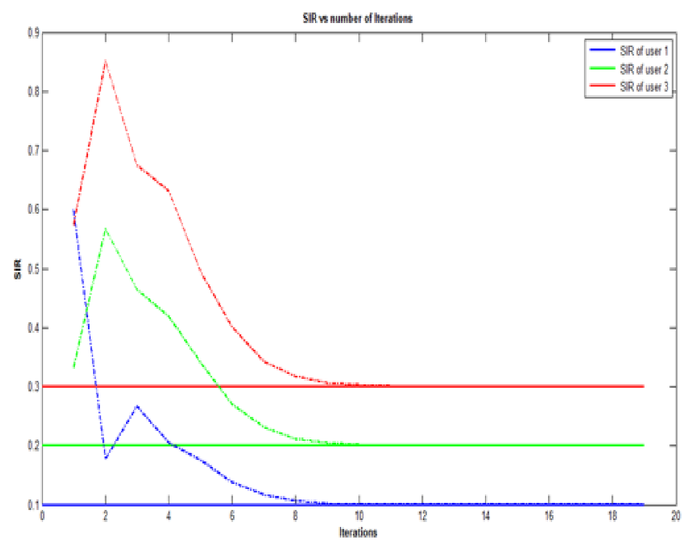

Figure 9: The signal-to-noise ratio and the number of iterations for 3 users

Thanks to the signal-to-noise ratio we have been able to test some parameters of Qos (CQI, SINR, TB) to see the influence of some factors on these parameters, we will present all our results on graphs, we could test.

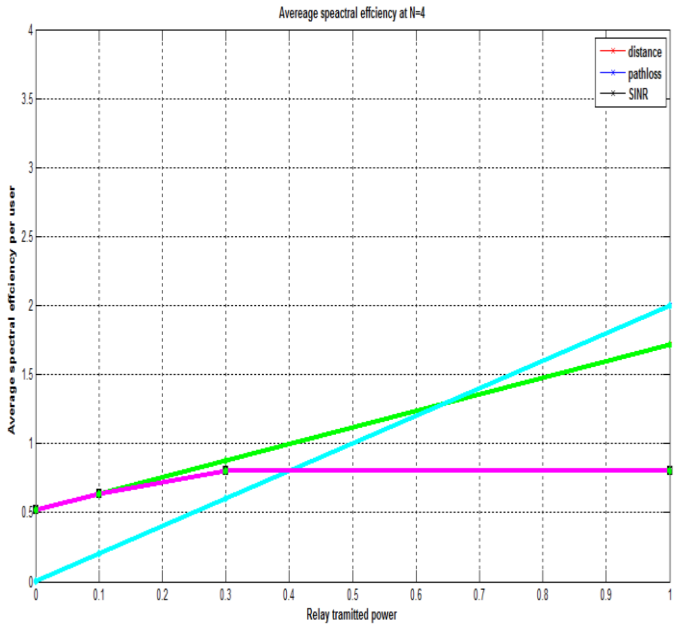

Figure 10: The average of spectral efficiency

On this graph we clearly see an excessive decrease in the SINR ratio, which is close to the value 0 at almost 1km distance separating the user (UE) and the antenna (eNodeB) while considering a fixed value of the noise at 148.947 dB , this is mainly due to signal attenuation, fading, scattering and multipathing.

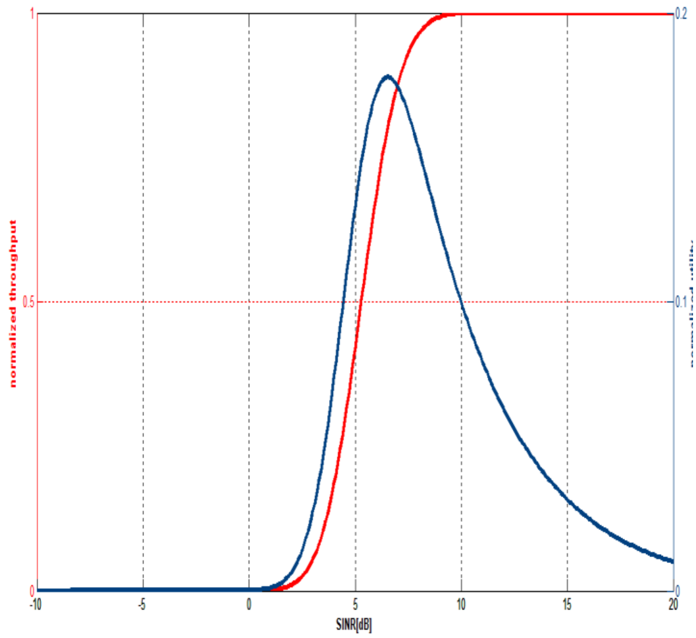

Figure 11: The throughput and the utility according to the signal-to-noise ratio

This graph tells us the quality of the channel and the number of transported block according to the distance separating the UE from the eNodeB, so we notice that the quality is better from 0 to 360m, after this value, a degradation is observed in staircase, this degradation and due mainly to the decrease of the intensity of the signal, the increase of the rate of binary error but also to the interferences.In this study, we focused our discussions, simulations and interpretations in the transmission part of the system with a particular focus on aspects of the transmission and signaling channel. It has been deduced that the quality of service indicates the reliability of the network by involving powerful parameters for the transmission, for that we have tested the evolution in the time and space of some existing parameters in the 4G networks namely: spectral efficiency, SINR (Signal Interference Noise Ratio), utility and throughput.

8. CONCLUSION

In this section we focus on the applicability and importance of game theory in networks. Many competitive situations exist in the context of networks:

Competition between operators or service providers regarding the type and quality of service offered as well as the pricing policy. The auctions play a very important role in networking. Operators must go through auctions for the allocation of radio resources that are often very expensive. On the other hand, auctions have penetrated the Internet through which internet users can sell or buy goods using auctions.

Knowledge of the characteristics of the Internet could bring benefits to this activity. On the other hand, there is a major research effort to analyze "artificial" competition situations, in the sense that, in reality, the individuals concerned behave in a cooperative manner. However, reflections related to these situations could lead to new protocols that are more robust and more decentralized (and therefore potentially simpler for scaling in large networks). Here are some examples, several versions of TCP (congestion control) protocols exist, they are all protocols that adapt to congestion and give up resources during congestion. We did not observe any non-cooperative behavior among Internet users that would result in the use of aggressive protocols. Power control in cellular networks has been studied extensively in the non-cooperative context. In reality, it seems that cell phone subscribers have not adopted non-cooperative behavior. We conclude that cooperative behavior is often adopted in networks, even when it seems that we can take advantage and get more playing alone. We explain this phenomenon by the fact that telecom services are probably sufficiently satisfactory (from the point of view of quality and price) for the benefits of non-cooperative behavior not to be worth the effort required to change technology to competition among users as possible.

ACKNOWLEDGMENT

We would like to thank the CNRST of Morocco (I 012/004) for support

REFERENCES

[1] Sara Riahi, Azzeddine Riahi,'' Energy Efficiency analysis in wireless systems by game theory '',5th international IEEE Congress on information Sciences and Technology IEEE CISt'18,Marrakech ,Morocco ;October 21-24,2018.

[2] Theodore L. Turocy, Bernhard von Stengel,'' Game Theory'', CDAM Research Report LSE-CDAM-2001-09, October 8, 2001.

[3] Rasmusen Eric , ''Games and Information: An Introduction to Game Theory'', 3rd edition. Blackwell, Oxford,2001.

[4] Levent Koc¸kesen, Efe A. Ok,'' An Introduction to Game Theory'', July 8, 2007.

[5] Thomas S. Ferguson, ''GAME THEORY'', University of California at Los Angeles 2012.

[6] Sylvain Sorin ,'' A First Course on Zero-Sum Repeated Games, Mathematiques & Applications '', Springer,2012

[7] Ronald J. Brachman, Tom Dietterich, '' ESSENTIALS OF GAME THEORY'' ,2007.

[8] Jaeok Park and Mihaela van der Schaar, '' The Theory of Intervention Games for Resource Sharing in Wireless Communications'', arXiv: 1101.3052v2 [cs.GT], Page No (1-29), 16 Jul 2011.

[9] Gaoning He & Samson Lasaulce, Yezekael Hayel, Merouane Debbah,'' A Multi-level Hierarchical Game-Theoretical Approach for Cognitive Networks'',2010.

[10] Tatiana Gvozdeva, Ali Hameed and Arkadii Slinko,'' Hierarchical Simple Games: Weightedness and Structural Characterization'',2004.

[11] COLIN F. CAMERER TECK-HUA HO JUIN-KUAN CHONG,'' A COGNITIVE HIERARCHY MODEL OF GAMES'', 2014.

[12] Sara Riahi ,Ali Elhore ,'' Estimation of QoS in 4th generation wireless systems'' , LOPAL '18 Proceedings of the International Conference on Learning and Optimization Algorithms: Theory and Applications, ISBN: 978-1-4503-5304-5,May 2018

[13] Sara Riahi, Azzeddine Riahi,'' Game theory for resource sharing in large distributed systems'', International Journal of Electrical and Computer Engineering IJECE, ISSN: 2088-8708 , vol 9 No 2, April 2019.

[14] Sara Riahi, Azzeddine Riahi,'' Optimal performance of the opportunistic scheduling in new generation mobile systems '', ICSDE'18 Proceedings of the 2nd International Conference on Smart Digital Environment, ISBN: 978-1-4503-6507-9 doi>10.1145/3289100.3289106,October 2018.

[15] Lopez DP, Chu XL, Athanasios VV, Claussen H. Power Minimization Based Resource Allocation for Interference Mitigation in OFDMA Femtocell Networks. IEEE Journal on Selected Areas in Communications. 2014; 32(2): 333–344.

[16] K. Cohen, A. Leshem and E. Zehavi, "Game Theoretic Aspects of the Multi-Channel ALOHA Protocol in Cognitive Radio Networks," in *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 11, pp. 2276-2288, November 2013.

[17] Wei Huang,'' Application of Game Theory in Wireless Communication Networks'', the faculty of graduate studies,Electrical and Computer Engineering, The University of British Columbia (Vancouver), Page No(1-160),February 2012.

[18] Hao Chen , Ying Liu , Jianfu Teng**,** " Optimal Threshold of LTE-Femtocell Network Based Bayes-Nash Equilibrium Theory ",TELKOMNIKA Indonesian Journal of Electrical Engineering Vol. 12, No. 12, December 2014, pp. 8246 ~ 8251 DOI: 10.11591/telkomnika.v12i12.6694

[19] Zhu HanDusit, NiyatoWalid, SaadTamer, BaarAreHjrungnes, "Game Theory in Wireless and Communication Networks: Theory, Models, and Applications,"*Game Theory in Wireless and Communication Networks: Theory, Models, and Applications1st*Cambridge University Press New York, NY, USA ©2012 ,ISBN:0521196965 9780521196963.

[20] Sara Riahi, Azzeddine Riahi,'' Game theory for resource sharing in large distributed systems'', International Journal of Electrical and Computer Engineering (IJECE), Vol. 9, No. 2, April 2019, pp. 1249~1257, ISSN: 2088-8708, DOI: 10.11591/ijece.v9i2.pp.1249-1257

[21] BADR BENMAMMAR , FRANCINE KRIEF,'' Game theory applications in wireless networks: A survey'', Applications of Information Systems in Engineering and Bioscience, ISBN: 978-960-474-381-0

[22] Sara Riahi, Ali El Hore, '' Fairness allocation and sharing of resources by game theory: modelization and algorithms'', ,IEEE-2018 International Conference on Control, Automation and Diagnosis (ICCAD'18) ,in proceeding ,2018

[23] MARTIN J. OSBORNE ,'' Publicly-available solutions for AN INTRODUCTION TO GAME THEORY", University of Toronto, Version 6: 2012-4-7,2012.

[24] C. Singhal, S. De, R. Trestian and G. M. Muntean, "Joint Optimization of User-Experience and Energy-Efficiency in Wireless Multimedia Broadcast," in *IEEE Transactions on Mobile Computing*, vol. 13, no. 7, pp. 1522-1535, July 2014.

[25] Game Theory for Wireless Sensor Networks: A Survey Hai-Yan Shi, Wan-Liang Wang, Ngai-Ming Kwok and Sheng-Yong Chen Sensors 2012, 12, 9055-9097; doi:10.3390/s120709055

[26] D. Lee, L. F. Gonzalez, J. Periaux. Hybrid-Game Strategies for multi-objective design optimization in engineering. Computers & Fluids 2011, 47 pp. 189-204

[27] Xin Ai , V. Srinivasan , Chen-khong Tham, Optimality and Complexity of Pure Nash Equilibria in the Coverage Game, IEEE Journal on Selected Areas in Communications, v.26 n.7, p.1170-1182, September 2008.

[28] Park, T.; Ryu, K.R. A Dual-Population Genetic Algorithm for Adaptive Diversity Control. IEEE Trans. Evol. Comput. 2010, 14, 865–884.

[29] Lee, D.; Gonzalez, L.F.; Periaux, J.; Srinivas, K.; Onate, E. Hybrid-Game Strategies for multi-objective design optimization in engineering. Comput. Fluids 2011, 47, 189–204.

[30] M. Zamarripa, A. Aguirre, C. Méndez. Integration of Mathematical Programming and Game Theory for Supply Chain Planning Optimization in Multi-objective competitive scenarios. Computer Aided Chemical Engineering. 2012, 30 pp. 402-406